

Percentages and Discounts Review

Basic Idea of Percentages

A percentage is simply a different way of saying a fraction out of 100.

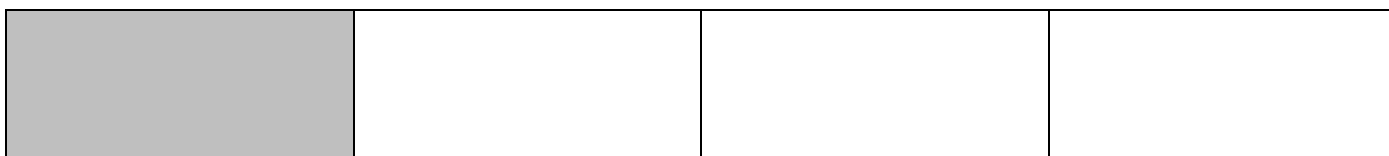
For example, say Farmer Fred had a flock of sheep that had 100 animals in it. If 3 of the sheep had black wool (as opposed to white), we would say that a **fraction** of $\frac{3}{100}$ sheep were black. Expressed as a **percentage**, we would say that **3%** of Farmer Fred's sheep were black.

Equivalent Fractions and Percentages

Of course, not every situation will be dealing with a group that has a total of 100 items so we need to be able to turn the fraction we are talking about into an equivalent fraction with a denominator (bottom figure) of 100.

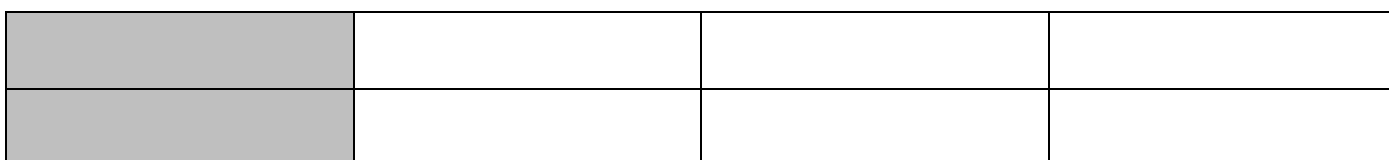
Say another farmer, Bill, had a smaller flock of sheep – 20 animals – and just one of them was black. We would say that $\frac{1}{20}$ sheep were black easily enough, but what would we say the percentage of sheep that are black would be?

Look at the following example first:



Here we would say that $\frac{1}{4}$ of the blocks are shaded.

However, if we split up the blocks into smaller pieces as below, we would now say that $\frac{2}{8}$ of the blocks are shaded.



As you can see, though the fraction is written differently, it is exactly the same proportion of blocks that are shaded – the fractions $\frac{1}{4}$ and $\frac{2}{8}$ are equivalent fractions. We can create an equivalent fraction by multiplying both the numerator (top number) and denominator by the same number (in this case, by 2).

In this example, if instead of multiplying by 2, we multiply the numerator and denominator by 25, the equivalent fraction would be $\frac{25}{100}$ and so we would say that 25% of the blocks are shaded.

Back to the Farm

If we now do the same thing with Farmer Bill's sheep situation, we would say that:

$$\frac{1}{20} = \frac{1 \times 5}{20 \times 5} = \frac{5}{100} \text{ or, } 5\%.$$

Practice Questions

Complete the following table:

| Fraction | Fraction as hundredths | Fraction as a percentage |
|-----------------|------------------------|--------------------------|
| $\frac{3}{10}$ | $\frac{30}{100}$ | 30% |
| $\frac{9}{10}$ | | |
| $\frac{1}{2}$ | | |
| $\frac{3}{4}$ | | |
| $\frac{17}{25}$ | | |

Word Question 1: If a school class has 25 students in it, and 14 are boys and 11 are girls, what are the percentages of boys and girls in the class?

Word Question 2: If two out of five people have curly hair, what percentage of people would be said to have curly hair?

Word Question 3 (challenge): If a dress costs \$200 at first but then is discounted by \$42, what would you say the percentage discount was?

Calculating Percentage Amounts, Discounts and Discounted Prices

Above, we were given two amounts – the number of a certain part of a group and the total number of the whole group - and then had to calculate the percentage that the part was of the whole. In many situations, we want to do the reverse – we are given the percentage of the part and the total of the whole and have to find the number of the part.

For example, if we are told that 25% of cattle in a herd of 200 are less than a year old, we can calculate that 50 of the cattle are less than a year old. Similarly, if it said that in a school of 350 students, that 18% got average marks of “B” or above, we can calculate that that would mean 63 students achieved these marks.

The way to do these calculations is to convert the percentage given to a decimal fraction and then multiply that decimal fraction by the total of the group.

Let's work through both of the examples above.

Example 1 – Cattle Herd

$$25\% = \frac{25}{100} = 0.25$$

$$\begin{array}{r} 0.25 \\ \times 200 \\ \hline 000 \\ 000 \\ \hline 05000 \\ 50.00 \end{array}$$

(work through normal multiplication technique ignoring the decimals and then putting them back at the end to get your final answer)

So, we have calculated that 50 cattle in the herd of 200, are less than a year old.

Example 2 – School Students

$$18\% = \frac{18}{100} = 0.18$$

$$\begin{array}{r} 0.18 \\ \times 350 \\ \hline 000 \\ 900 \\ \hline 05400 \\ 63.00 \end{array}$$

So, here we have calculated that 63 students out of the 350 in the school got average marks of "B" or above.

Discounts and Discounted Prices

When the price of something is discounted by a percentage, we can calculate the amount that has been taken off by the same method as above. Then, we can find the actual price to be paid by taking away the discount amount from the original starting price.

Example Problem

What is the discount and final price of a dress that originally was \$280 but has been discounted by 30%?

$$\text{First step as above: } 30\% = \frac{30}{100} = 0.30$$

$$\begin{array}{r}
 0.30 \\
 \times 280 \\
 \hline
 000 \\
 2400 \\
 \hline
 06000 \\
 \hline
 84.00
 \end{array}$$

So, the dress has been discounted by \$84.

The new price is then $\$280 - \$84 = \underline{\$196}$.

Final Practice Questions

- Q1. If 60% of the rooms in a 200 room unit block need new carpet, how many rooms would that be?
- Q2. If 85% of the 160 Year 6 students in a school will be continuing to Year 7 in the same school, how many will that be?
- Q3. A farm of 200 hectares has 90 hectares infested with weeds. What percentage of the farm has the infestation problem?
- Q4. A cheap car costing \$4,000 has been sitting unsold in the second-hand car yard for too long so the manager has decided to offer a discount of 15% to try to sell it quickly. What price is he asking for it now?
- Q5. A pair of shoes originally priced at \$90 has been discounted by 25%. What is the new price of the shoes?

PS – Estimating Discounts and Final Prices

Estimating discounts in your head is useful when a shop shows the original price on the price tag and only puts the discounted price on the sale docket, so that you can estimate if you have been given the full discount advertised.

With the example of the dress discounted above, one way to estimate the discount in your head, is to work what a 10% discount would be – in this case \$28 (being $\$280 \div 10$ since $10\% = 0.1 = \frac{1}{10}$). This is close to \$30. Since the discount is 30%, it will be 3 times the amount that a 10% discount would be - so an estimate of the discount you should receive would be $3 \times \$30 = \90 .

Now $\$280 - \90 is something you may be able to work out in your head as well, being 10 times $28 - 9 (=19)$. So, in the end, your estimate of the discounted price would be $10 \times 19 = \$190$. When you see the docket showing \$196, you can tell it is at least close to being correct as you estimated \$190, and that was based on a slight overestimate of the discount (by rounding \$28 up to \$30 in the calculations above).